UNIVERSITAS
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# Pemodelan Stokastik dan Aplikasinya

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# Mathematics is Everywhere!



- *Weather forecast*


Peluang dari cuaca esok hari hanya dipengaruhi oleh cuaca pada hari ini!

Proses Stokastik (*Markov Chain*)

# Stochastic vs Deterministic Processes



1. A **stochastic process** $X = \{X_t, t \in T\}$ is also known as **random process**.

   $X_t$ : *state* dari proses pada waktu *t*.

   State space : himpunan seluruh kemungkinan nilai $X_t$

- There is some **indeterminacy**: even if the initial condition is known, there are several (often infinitely many) directions in which the process may evolve.
- In many stochastic processes, the movement to the next state **depends on only the current state**, and is independent from prior states or values the process has taken ➡ ***Markov Chain***

  (probabilitas transisi, *memoryless*)

$$\Pr\{X_{n+1} = j \mid X_n = i, X_{n-1} = i_{n-1}, \ldots, X_1 = i_1, X_0 = i_0\}$$
$$= \Pr\{X_{n+1} = j \mid X_n = i\} = P_{ij}$$

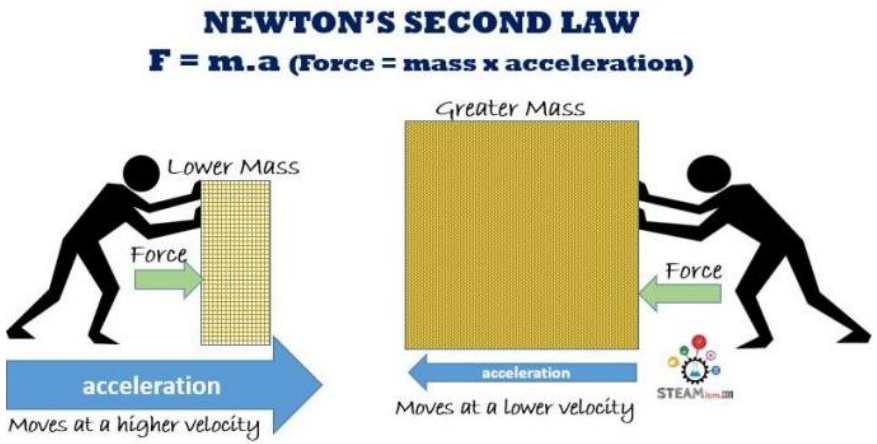
2. In **deterministic process** if the initial point is known, the next step is predictable.

# Stochastic vs Deterministic Processes



- In deterministic processes, the outcome can be predicted exactly in advance
  - Eg. Force = mass x acceleration. If we are given values for mass and acceleration, we exactly know the value of force

- In random processes, the outcome is not known exactly, but we can still describe the *probability distribution* of possible outcomes
  - Eg. 10 coin tosses: we don't know exactly how many heads we will get, but we can calculate the probability of getting a certain number of heads




# *Continued*... (Weather forecast)



Peluang dari cuaca esok hari hanya dipengaruhi oleh cuaca pada hari ini!

- *State*: *hujan vs tidak hujan*
- $n$ : hari ke-$n$
- $X_n$ : realisasi cuaca pada hari ke–$n$ *dengan*

$$X_n = \begin{cases} 1 \text{ , jika hari ke} - \text{n hujan} \\ 0 \qquad\qquad\qquad \text{, yang lain} \end{cases}$$

Rantai Markov Diskrit

Qn: Diagram transisi ???

# Diagram Transisi



Peluang cuaca esok hari hanya dipengaruhi oleh cuaca pada hari ini

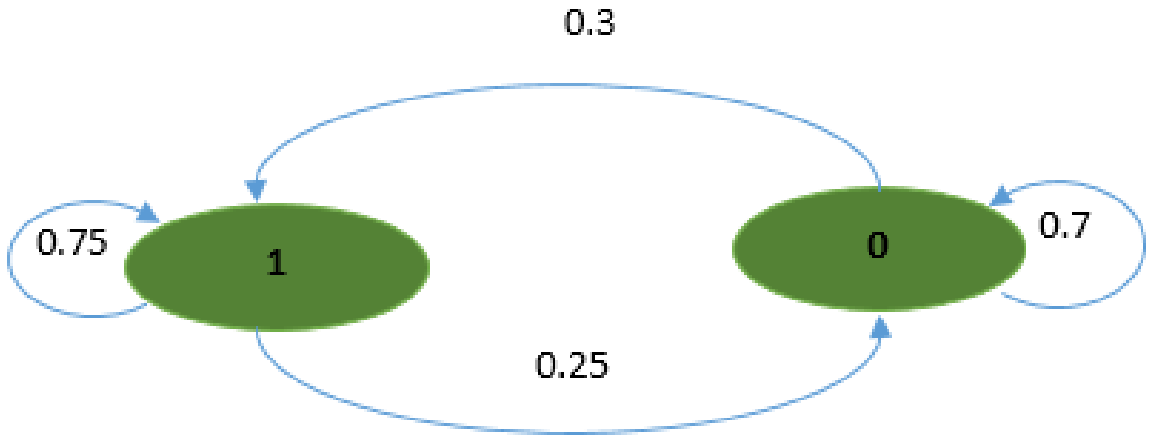
- Hari ini hujan → 75 % kemungkinan besok hujan
  25 % kemungkinan besok tidak hujan

- Hari ini tidak hujan → 30 % kemungkinan besok hujan
  70 % kemungkinan besok tidak hujan

***Diagram Transisi ???***

Hujan = 1

Tidak hujan = 0

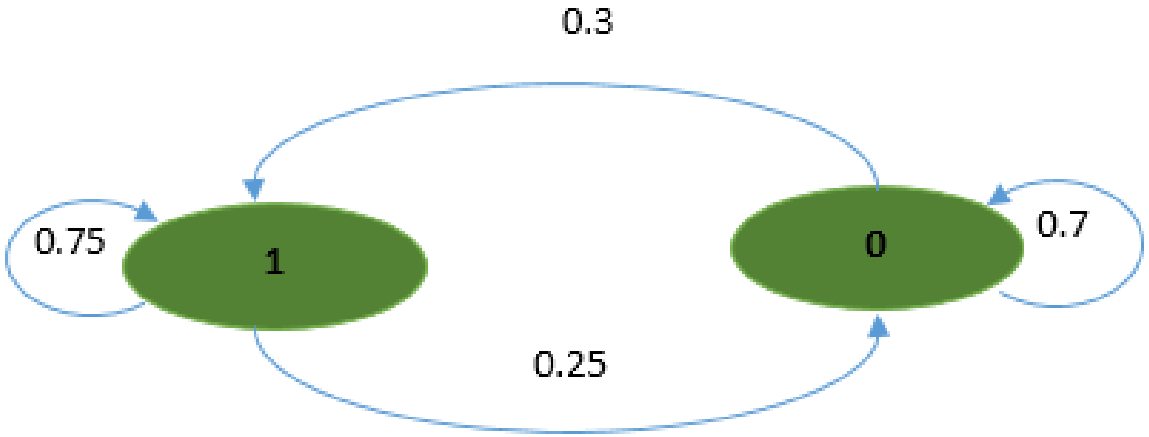



Qn: Matriks peluang transisi ???

# Matriks peluang transisi



$$P = \begin{pmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{pmatrix} = \begin{pmatrix} 0.7 & 0.3 \\ 0.25 & 0.75 \end{pmatrix}$$




## Bentuk umum dari matriks peluang transisi

$$\mathrm{P} = \begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} & \cdots \\ P_{10} & P_{11} & P_{12} & P_{13} & \cdots \\ P_{20} & P_{21} & P_{22} & P_{23} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \\ P_{i0} & P_{i1} & P_{i2} & P_{i3} & \cdots \\ \vdots & \vdots & \vdots & \vdots & . \end{bmatrix}$$

$P_{ij}$ : peluang transisi satu langkah dimana $X_{n+1}$ berada pada state $j$ dengan syarat $X_n$ berada pada state $i$

# *Continued*... (Weather forecast)



Question:

Peluang 3 hari ke depan turun hujan berturut-turut, jika diketahui hari ini tidak hujan ???

Diketahui:

Peluang hari ini tidak hujan = 1,

$$\text{i.e., } P(X_0 = 0) = P_0 = 1,$$

sehingga peluang hari ini hujan = 0,

$$\text{i.e., } P(X_0 = 1) = P_1 = 0.$$

Jadi,

$$P(X_0 = 0,\ X_1 = 1,\ X_2 = 1,\ X_3 = 1) = P_0 P_{01} P_{11} P_{11}$$

$$= (0.3)(0.75)(0.75) = 0.16875 \approx 17\%$$

- Peluang Gabungan

  Secara umum,

$$\Pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \cdots, X_{n-1} = i_{n-1}, X_n = i_n\} = p_{i_0} P_{i_0 i_1} P_{i_1 i_2} \cdots P_{i_{n-1}, i_n}$$

- Transisi *n* langkah

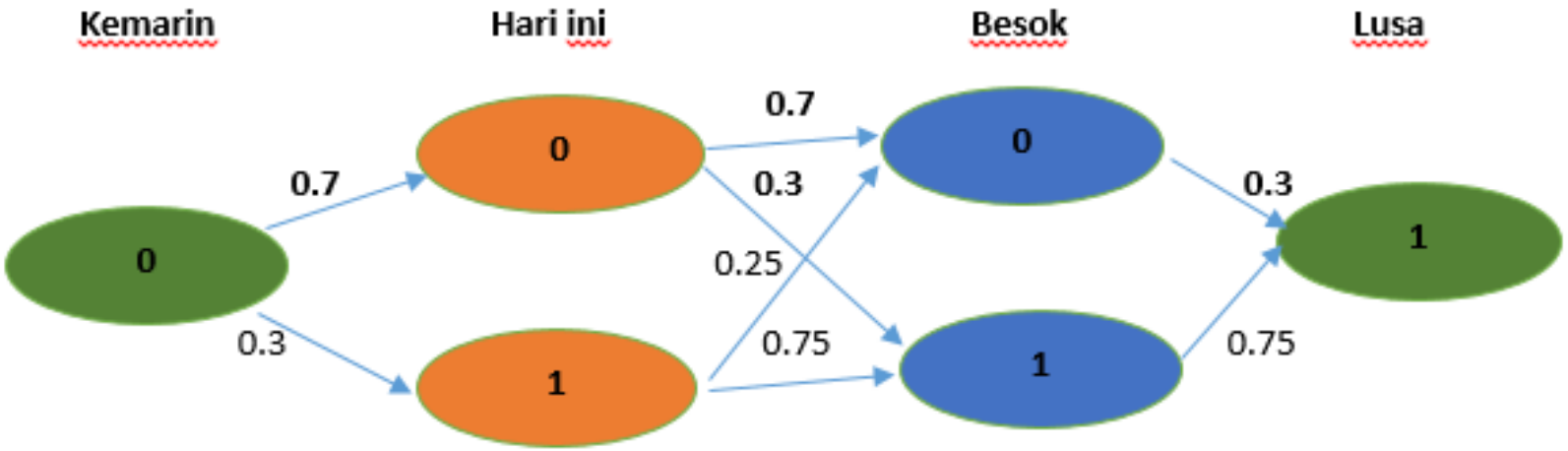
  *Chapman-Kolmogorov equation:* $P_{ij}^{(n)} = \sum_{k=0}^{\infty} P_{ik} P_{kj}^{(n-1)}$

# *Continued*... (Weather forecast)



Question:

Peluang bahwa lusa turun hujan jika kemarin tidak hujan???




By Chapman-Kolmogorov Eqn.

$$P_{01}^{(3)} = P_{00}P_{01}^{(2)} + P_{01}P_{11}^{(2)}$$

$$= P_{00}(P_{00}P_{01} + P_{01}P_{11}) + P_{01}(P_{10}P_{01} + P_{11}P_{11}) = 0.9525 \approx 95\%$$